THE OPTIMAL CONTROL OF THE MULTIDIMENSIONAL SYSTEM AT STOCHASTIC EXCITATION

Alex Podzorov, Alexander Gorobtsov, Mikhail Lyashenko

Volgograd State Technical University
Auto-Tractor Fakulty
Department of Higher Mathematics
Lenin avenue 28, 4000131 Volgograd, Russia
tel.: +7 (8442) 291435, fax:+7 (8442) 248182
e-mail: legionerpav@gmail.com, gorobtsov@avtlg.ru

Abstract

The conflict between ride and handling in a conventional passive suspension system is extremely difficult to solve. Therefore, a considerable amount of work has been carried out over in this area. Over recent years the progress in actuator and microelectronics technology has made controllable suspension systems feasible. These systems are designed to reduce the drivers' exposure to harmful vibration, as well as to improve the handling properties of the vehicle. Due to widespread use of vehicles, a myriad of different control schemes and algorithms can be found in the literature for these systems. The problem of control by a suspension system of a vehicle on the basis of a principle of maximum Pontrjagin is considered in this article. The expressions for definition of laws of optimal control of damping and springing forces in elements of a suspension system for measure of ride of a vehicle are received. The results of computer modelling of process of work of a semi-active suspension system of the bus with the controlled shock-absorber for a movement on stochastic microprofile in system of modelling of dynamics of solid and elastic bodies - FRUND are presented. This study demonstrated that semi-active suspensions could effectively improve vehicle ride. The results of computer experiment have shown advantages of a semi-active suspension.

Keywords: *optimal control, shock-absorber, stochastic excitation, semi-active suspension, multidimensional system*

1. Introduction

The problem of optimal control by multidimensional dynamic systems is actual one for such objects as suspension systems of vehicles. Methods of optimal control underlie making of algorithms of operation of the technical systems for minimization of indexes of dynamics of the object. The mathematical model of multidimensional oscillating dynamic system possesses a sufficient generality for its usage in the capacity of models of any dynamic system, for example in the capacity of models of a suspension system of the car.

The car suspension fulfils some the important functions, handleability, stability, ride comfort and other properties of the car depend on its construction and operating characteristics. [2, 3, 7].

Among controllable suspension systems semi-active suspensions are receiving considerable attention because of their low cost and competitive performance compared with their active alternatives. Active suspensions have the ability to add energy into the system, as well as store and dissipate it. By means of an adjustable damper, a semi-active suspension is only capable of storing and dissipating energy. According to the damper configurations used, semi-active suspensions can be classified into three categories: dampers with a controllable orifice, dampers with controllable fluids and friction dampers. Dampers with a controllable orifice are the most common forms of semi-active suspensions and commercial products are already available. Currently ZF Sachs provides CDC (continuous damping control) semi-active suspension units [9] which have been successfully applied on some vehicle models. Dampers with controllable fluids are often known as electrorheological (ER) or magnetorheological (MR) dampers depending on which rheological fluid is used [11]. The properties of these fluids are determined by polarisable particles within a nonconductive carrier fluid. When these particles are polarised, the fluid can become very viscous and difficult to move. Despite the advantages of fast response and almost no wear, these dampers are sensitive to contamination, which limits their applications in mass production vehicles. In addition the fluid viscosity is temperature dependant and this conflicts with the controllability of these devices. Friction dampers are devices using dry friction forces [8]. However, it is argued that the choice of using dry friction as a mean of achieving damping forces is of little practical use in automotive applications because of harshness and wear problems.

At present there are control systems of suspension which work on the basis of electronic circuits or controllers. Such systems demand equipment of suspension by sensors and actuators. The controller registers the sensors reading and will convert them to certain commands to actuators which implement indicated values of parameters of a suspension system [4, 6].

In this paper the control system by multidimensional system on the basis of a principle of maximum Pontrjagin is presented. In the capacity of control parameter the damping rate of the chock-absorber has been used. Damping rate varied stepwise under the determined law.

2. Sky-hook control of the suspension of a vehicle

Firstly introduced by Karnopp and Crosby [10], skyhook damping is a widely used concept in the annals of controllable suspensions research. The ideal skyhook is a fictitious suspension where the suspension damper is assumed to be connected from the sprung mass M to a point that might be considered to be sliding in a perfectly straight groove or channel at the same speed as the car, Fig. 1(a).

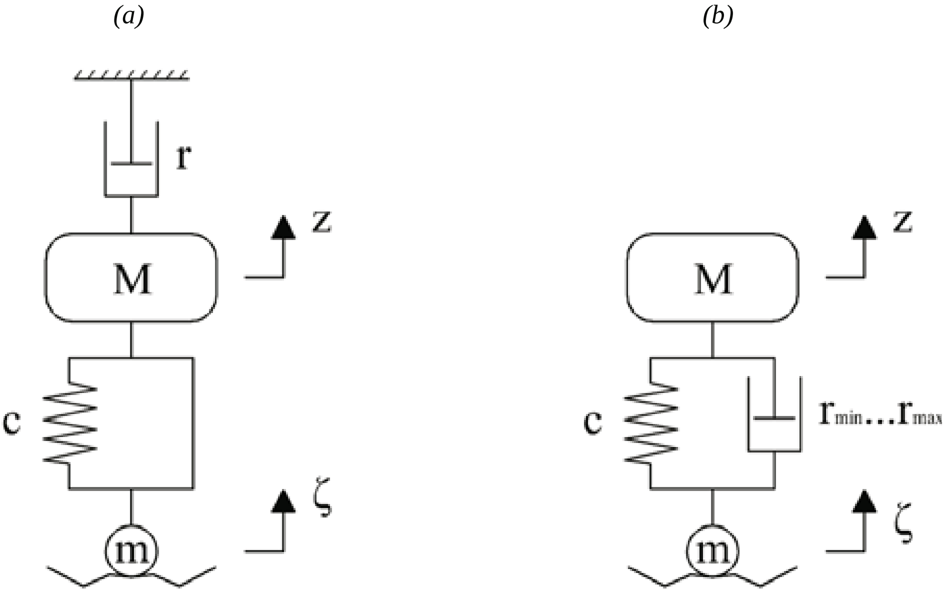

Fig. 1. Ideal sky-hook damping 1(a) and practical implementation of sky-hook damping 1(b)

In reality, it is impossible or impractical to arrange such a system. Damper should be established in parallel to springing element and connect sprung mass M and unsprung mass m, as is shown in Fig. 1 (b). The desired damping force should be created with a variable damping rate. It should be noted that since a semi-active suspension is only capable of dissipating energy, the control action is applied only when the control force is opposite to the direction of suspension relative velocity.

This strategy can be understood as follows. If the ideal skyhook configuration has $\dot{z} > 0$, namely the velocity of the sprung mass is upwards, the ideal skyhook damper will be in compression and exert a downward force on the sprung mass. An actual damper connected between the sprung and unsprung masses can exert a downward force on the sprung mass only if it is in extension. Thus, if the sprung mass is moving up faster than the unsprung mass or the unsprung mass is moving downward, the actual damper will be in extension and the quantity $\dot{z}-\dot{\zeta}>0$, therefore the product $\dot{z}(\dot{z}-\dot{\zeta})>0$ will also be positive. In this case, the suspension is set to the 'hard' setting. If on the other hand, the unsprung mass is moving upward faster than the sprung mass, then $\dot{z}-\dot{\zeta}<0$ and $\dot{z}(\dot{z}-\dot{\zeta})<0$. In this case, the damper will be in compression and will exert an upward force on the sprung mass. This is not an appropriate characteristic to emulate the ideal skyhook. As a result, under these conditions the suspension is set to the smallest possible damping rate to minimise the compressive force.

In this principle, the two-stage control logic was designed as:

$$r=\begin{cases} r_{\min}, & \dot{z}(\dot{z}-\dot{\zeta})<0 \\ r_{\max}, & \dot{z}(\dot{z}-\dot{\zeta})>0 \end{cases}, \tag{1}$$

where:

$r_{\min}$, $r_{\max}$ - minimum and maximum values of damping rate,
$\dot{z}$ - velocity of sprung mass,
$\dot{\zeta}$ - velocity of unsprung mass.

3. Law of optimal control of multidimensional system

The general view of the equations of movement of multidimensional system, in the assumption of fine movements, registers in a matrix kind as follows:

$$M\ddot{x}+CU_{\lambda}+DU_{\dot{\lambda}}+f(\dot{x},x,t)=0, \tag{2}$$

where:

M - mass matrix ($M=diag(m_1,m_2,...,m_n)$),
x - vector of the generalized coordinates of bodies of system, dimension n ($x^T=(x_1,...,x_n)$),
U_{λ} - vector function switching springs ($U_{\lambda}^T=(u_{c1}\lambda_1,...,u_{cn1}\lambda_{n1})$),
$U_{\dot{\lambda}}$ - vector function switching dampers ($U_{\dot{\lambda}}^T=(u_{d1}\dot{\lambda}_1,...,u_{dn2}\dot{\lambda}_{n2})$),
$n1$ - number of springs,
$n2$ - number of dampers,
$u_{ci}=u_{ci}(t)$ - function of control by spring element of a suspension system which can accept two fixed values,
$u_{dj}=u_{dj}(t)$ - function of control by damper of a suspension system which can accept two fixed values,
$C=()_{n\times n1}$ - matrix which describes springs rate and coefficients of reduction of springing forces to directions of degrees of freedom,
$D=()_{n\times n2}$ - matrix, which describes dampings rate and coefficients of reduction of damping forces to directions of degrees of freedom,
λ_i - relative movement in the i-th springing element of a suspension,
$\dot{\lambda}_j$ - relative velocity in the j-th damping element of a suspension,

$f(\dot{x}, x, t)$ - vector function of other forces and the external excitation.

The problem of optimal control is formulated as follows. Find a switching law, which provides the minimum of a functional of a quality:

$$I = \int_0^t \bar{f}(x, \dot{x}) dt \,. \tag{3}$$

The decision is searched on the basis of a maximum principle [1]. For this purpose we represent system of the equations (2) in the form of system of the first order. We carry out replacement of variables:

$$y^T = (x_1, ..., x_n, \dot{x}_1, ..., \dot{x}_n) \,. \tag{4}$$

Now the equations (2) can be written in a kind:

$$M^* \dot{y} + C^* U_\lambda^* + D^* U_{\dot{\lambda}}^* + f^*(y,t) = 0 \,, \tag{5}$$

where:

$$M^* = \begin{pmatrix} E & 0 \\ 0 & M \end{pmatrix}, \; C^* = \begin{pmatrix} 0 \\ C \end{pmatrix}, \; D^* = \begin{pmatrix} 0 \\ D \end{pmatrix}, \; f^*(y,t) = \begin{pmatrix} 0 \\ f(y,t) \end{pmatrix}.$$

From the equations (5):

$$\dot{y} = -M^{*-1}(C^* U_\lambda^* + D^* U_{\dot{\lambda}}^* + f^*(y,t)) \,. \tag{6}$$

The functional of a quality is accepted in a kind:

$$I = \int_0^t (By)^2 dt \,, \tag{7}$$

where:

B - some a row matrix of constant coefficients.

For the system (6) Hamiltonian can be represented as [1]:

$$H = \psi \dot{y} + \psi_{2n+1}(By)^2 = \psi \dot{y}^*(y) + \psi \dot{y}^{**}(u, y) + \psi_{2n+1}(By)^2 \,, \tag{8}$$

where:

$\dot{y}^*(y)$ - set of the functions which are not containing controlling function,

$\dot{y}^{**}(u, y)$ - set of the functions which are containing controlling function (i.e. functions $c_i u_{ci} \lambda_i + d_i u_{di} \dot{\lambda}_i$),

$\psi_{2n+1} =$ -1,

ψ - vector of conjugate variables, which are determined from the differential equations of a kind:

$$\dot{\psi} = -J\psi - \psi_{2n+1}K\,, \tag{9}$$

$$J = \begin{pmatrix} \frac{\partial \dot{y}_1}{\partial y_1} & . & . & \frac{\partial \dot{y}_{2n+1}}{\partial y_1} \\ . & . & . & . \\ . & . & . & . \\ \frac{\partial \dot{y}_1}{\partial y_{2n}} & . & . & \frac{\partial \dot{y}_{2n+1}}{\partial y_{2n}} \end{pmatrix}, K = \begin{pmatrix} \frac{\partial \bar{f}}{\partial y_1} \\ . \\ . \\ \frac{\partial \bar{f}}{\partial y_{2n}} \end{pmatrix}. \tag{10}$$

The system of differential equations (9) is solved with initial conditions defined by transversality conditions [1]:

$$(\psi, y) = 0\,. \tag{11}$$

I.e. with zero initial conditions. For any entry conditions, y_0 one of admissible values ψ_0 ($\psi_0 = 0$).

The Hamiltonian into which enters controlling functions, accepts a maximum provided that summands $\psi\,\dot{y}^{**}(u\,,\,y)$, $\psi\dot{y}^{*}(y)$ accept the maximum values, i.e. for separate summands of this expression of a condition for control function is possible to write as follows:

$$u = \begin{cases} u_{\max},\ \psi_i \dot{y}_i^{**} > 0 \\ u_{\min},\ \psi_i \dot{y}_i^{**} < 0 \end{cases}. \tag{12}$$

It is necessary to identify signs of conjugate functions for use of the expression (12). The values of conjugate functions are determined from the system of differential equations (9). Exact integration of system (9) in the general case is impossible. The solution of (9) will be unstable, as the matrix J will comprise positive-definite matrix C and D, with negative signs, therefore, the roots of the linearized characteristic equation will have a positive real part. The signs of second summands of right side (9) will determine the signs of the first derivatives of ψ. Therefore, we can approximately consider that, with zero initial conditions, the sign of conjugate functions in the infinitely small initial interval of time will coincide with the sign of second summand in equation (9):

$$sign(\psi_i) = sign\left(\frac{\partial \bar{f}}{\partial y_i}\right). \tag{13}$$

Then the condition of optimal control (12) for a small interval will register in a kind:

$$u = \begin{cases} u_{\max},\ b_i \dot{y}_i\, \dot{y}_i^{**} > 0 \\ u_{\min},\ b_i \dot{y}_i\, \dot{y}_i^{**} < 0 \end{cases}. \tag{14}$$

If to extend a condition (14) to all time intervals, we will receive simple algorithm of management. The strict substantiation (14) is impossible, however this expression supposes obvious physical interpretation, and can be used for the subsequent analysis.

Criterion (7) and conditions of switching (14) allow defining conditions of an optimal control for system of any dimension with any quantity of controls.

4. Results of computer modelling of a semi-active suspension of the bus

A computational experiment for check of efficiency of laws of management (14) has been made. Calculations were spent in system of modelling of dynamics systems of solid and elastic bodies - FRUND [5]. The multidimensional model of the bus was used, Fig. 2.

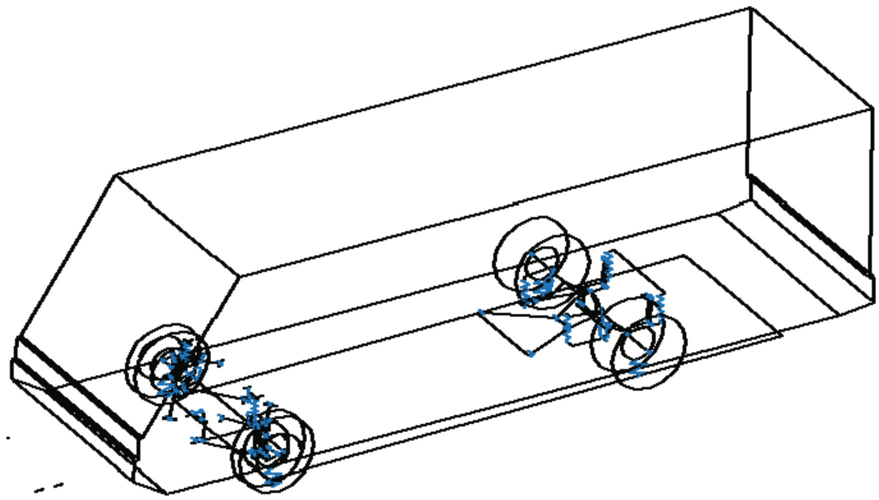

Fig. 2. Multidimensional model of the bus

Modelling of movement of the bus with semi-active and passive suspension on a stochastic microprofile with constant speed has been carried out. In the capacity of control parameter the damping rate of the chock-absorber has been used. Damping rate varied stepwise. The criterion of change of damping rate was a following kind:

$$y = \dot{z}\dot{\lambda}\,, \tag{15}$$

where:

$\dot{z}$ - vertical velocity of a body over the shock-absorber,

$\dot{\lambda}$ - strain rate of the shock-absorber.

The stochastic excitation represented the experimental realization of road of autoproving ground NAMI (Central Scientific Research Automobile and Engine Institute). For demonstration of a principle of switching between maximum and minimum value of damping rate of a damper had been constructed a time realization of change of vertical velocity of a body over the front damper, strain rate of the front damper and control signal. The given realization is presented on Fig. 3.

If the product of the vertical velocity of the body above the damper and the strain rate of the damper greater than zero, then stiff characteristic of damper, if less than zero then soft characteristics of damper.

The spectrums of vertical accelerations on a place of the driver at movement with a velocity of 60 km/h have been constructed for comparison of efficiency of a passive suspension and semi-active suspension, Fig. 4. First graph corresponds a passive suspension with soft damper (factor of aperiodicity $\psi = 0{,}1$), second a passive suspension with stiff damper (factor of aperiodicity $\psi = 0{,}7$), third a semi-active suspension with the variable characteristic of the damper. Resistance of the controlled damper varied from minimum ($\psi = 0{,}1$) to maximum ($\psi = 0{,}7$) according to changes of the level of control signal.

As can be seen from the graphs of the spectrums of vertical acceleration, semi-active suspension has advantages compared with passive suspension, equipped by standard dampers of different damping rate.

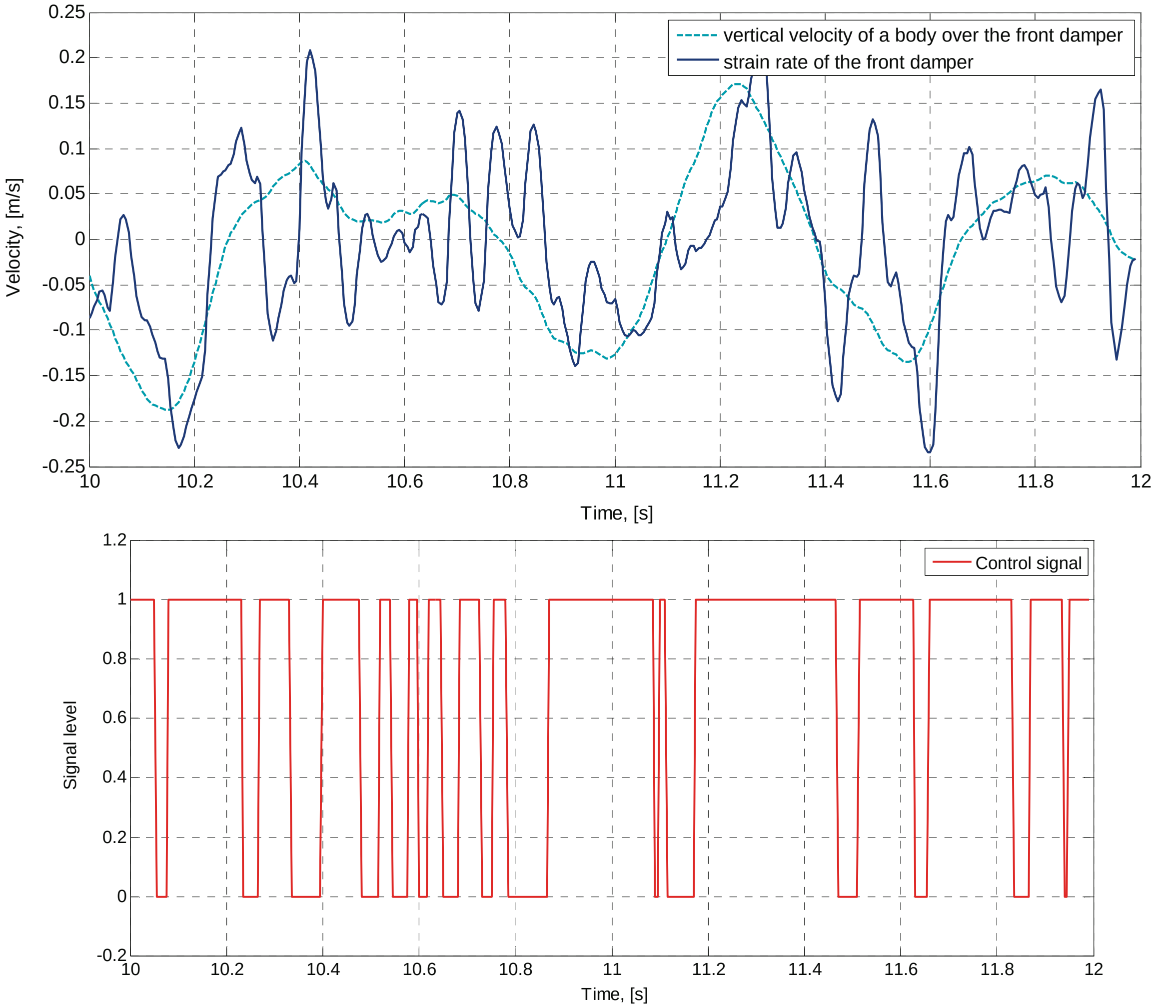

Fig. 3. Time realization

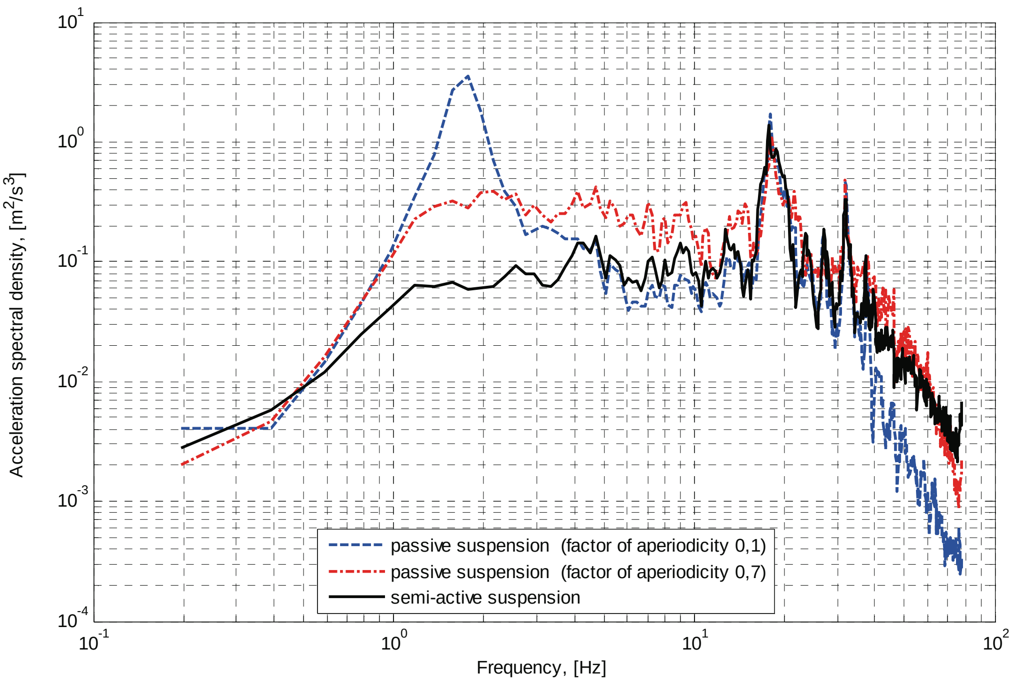

Fig. 4. Spectrums of vertical accelerations on a place of the driver

Root-mean-square accelerations on a place of the driver make in a range of frequencies of 0-3 Hz - 1,46 m/s^2, 0,79 m/s^2, 0,38 m/s^2, in a range of frequencies of 3-15 Hz - of 0,97 m/s^2, 1,59 m/s^2, 1,07 m/s^2, in a range of frequencies of 15-40 Hz - of 2,05 m/s^2, 2,09 m/s^2, 2,12 m/s^2 for a passive suspension with soft damper, for a passive suspension with stiff damper and for a semi-active suspension accordingly.

5. Conclusions

The problem of control by a suspension system of a vehicle on the basis of a principle of maximum Pontrjagin has been considered in this article. A method for obtaining optimal discrete control laws by multidimensional systems with stochastic excitation has been proposed. The three-dimensional model of the bus has been developed in system of modelling of dynamics of solid and elastic bodies - FRUND for check of effectiveness of offered laws of control. The numerical experiment has been spent on mathematical model of a semi-active suspension system of vehicle at a stochastic excitation from road. In the capacity of control parameter the damping rate of the chock-absorber has been used. Damping rate varied stepwise, accepting two fixed values. The stochastic excitation represented the experimental realization of road of autoproving ground NAMI. A comparison of a semi-active suspension and passive suspension, equipped by standard dampers of different damping rate for measure of ride has been spent. The results of computer experiment have shown advantages of a semi-active suspension. The method of deriving of laws of optimum control by multidimensional oscillating systems can be a bottom for creation mechanically concerning simple semi-active systems of vibroprotection.

References

[1] Boltyanskiy, V. G., *Matyematichyeskiye myetodi optimal'nogo oopravlyeniya*, Moskva, 1968.

[2] Borisov, S. V., Arhipov, A. I., Osipov, V. I., *Podvyeska avtomobilya, Oochyeb. Posobiye*, MADI TOo, Ch.1, Moskva, 1995.

[3] Voronov, A. A., Oostoychivost', oopravlyayemost', nablyoodayemost', Moskva, 1979.

[4] Gyenkin, M. D., Yelyezov, V. G., Yablonskiy, V. V., *Myetodi oopravlyayemoy vibrozashshiti mashin*, Moskva, 1985.

[5] Gorobtsov, A. S., Solodyenkov, S. V., A*lgoritmi chislyennogo intyegrirovaniya ooravnyeniy dvizhyeniya sistyemi tyel s mnozhityelyami Lagranzha*, Mashinostroyeniye i inzhyenyernoye obrazovaniye, №3 pp. 20-27 2005.

[6] Zhdanov, A. A., Lipkyevich, D. B., *AdCAS - sistyema avtonomnogo adaptivnogo oopravlyeniya aktivnoy podvyeskoy avtomobilya,* Troodi Institoota sistyemnogo programmirovaniya, Tom 7, Noviye podhodi v nyeyronopodobnih i osnovannih na znaniyah sistyemah, pp. 119-159, Moskva, 2004.

[7] Pavlyook, A. S., P*odvyeski lyegkovih avtomobilyey, Oochyeb. Posobiye*, Alt. gos. tyeh. oon-t im. I. I. Polzoonova, Barnaool, 1996.

[8] Guglielmino, E., Stammers, C. W., Edge, K. A., *Robust force control in electrohydraulic friction damper systems using a variable structure scheme with non linear state feedback*, Internationales Fluidtechnisches Kolloquium, Dresden, 2000.

[9] http://www.zf.com.

[10] Karnopp, D. C., Cosby, M. J., Harwood, R. A., *Vibration control using semi-active force generators*, Engineering for Industry, pp. 619-626, 1974.

[11] Kruger, W. R., Vaculin, O., Spieck, M., *Evaluation of Active Damping for Reduction of Noise, Vibration and Motion of Ground Vehicles by Multibody Simulation,* RTA/AVT Symposium on Habitability of Combat and Transport Vehicles, Noise, Vibration and Motion, Prague, 2004.